Algebraic Laws for Regular Expressions

• Associativity
  – \((L + M) + N = L + (M + N)\)
  – \((LM)N = L(MN)\)

• Commutativity
  – \(L + M = M + L\)
  – \(LM \neq ML\)

• Identities and Annihilators
  – \(\phi + L = L + \phi = L\)
  – \(\epsilon L = L = L\)
  – \(\phi L = L\phi = \phi\)
Algebraic Laws for Regular Expressions

• Distributive Laws
  – $L(M + N) = LM + LN$
  – $(M + N)L = (M + N)L$

• Idempotent Law
  – $L + L = L$

• Laws Involving Closures
  – $(L^*)^* = L^*$
  – $\phi^* = \epsilon$
  – $\epsilon^* = \epsilon$
  – $L^+ = LL^* = L*L$
  – $L^* = L^+ + \epsilon$
Test Regular Expression Law

• Test whether $E = F$:
  – Convert $E$ and $F$ to concrete regular expressions $C$ and $D$, respectively, by replacing each variable by a concrete symbol
  – Test whether $L(C) = L(D)$. If so, then $E = F$ is a law, and if not, then it is not a law.

• This only works for the three basic operations. This may not work for additional operations.
Pumping Lemma

Let $L$ be a regular language. Then, there is a constant $n$ such that if $z$ is any string in $L$, and $|z| \geq n$, there exist three strings $u$, $v$, $w$ so that

- $z = uvw$
- $|uv| \leq n$
- $|v| \geq 1$
- for all $i \geq 0, uv^i w$ is in $L$. 
Applications of Pumping Lemma

- Pumping lemma is mainly used to prove some language is not a regular language.
  1. Select the language $L$ you wish to prove nonregular.
  2. Pick up any integer $n$.
  3. Select a string $z$ in $L$. $|z|$ must be larger than $n$.
  4. Break $z$ into $u$, $v$, and $w$ in all possible ways so that $|uv| \leq n$ and $|v| \geq 1$.
  5. Prove $uv^iw$ for a given $i$ for example 2 is not in $L$. From pumping lemma, $L$ is not a regular language.
Example

1. Prove $a^ib^i$ is not a regular language.
2. Assume that $a^ib^i$ is accepted by an FA with $n$ states.
3. Let’s consider the string $a^nb^n$.
4. For any $u$, $v$, $w$ such that $|uv| \leq n$ and $|v| \geq 1$. We have $u = a^l$ and $v = a^m$ ($m \geq 1$) and $w = a^kb^n$ and $l + m + k = n$.
5. $uv^2w = a^l a^m a^m a^k b^n = a^{l+2mk} b^n$ is not in $L$, because $l+2m+k > n$. According pumping lemma, $L$ is not a regular language.
Closure Properties of Regular Languages

• A class of languages is closed under a particular operation if application of this operation to languages in this class results in a language also in this class.

• Theorem:
  The regular languages are closed under union, concatenation, and Kleene closure.

• Theorem:
  The regular languages are closed under complementation.

• Theorem:
  The regular languages are closed under intersection.