CS5104 Assignment 4  
Due: Wednesday, April 7

1. Consider the CFG

   \[ S \Rightarrow aX \]
   \[ X \Rightarrow aX|bX|\varepsilon \]

   What is the language this CFG generates?

Solution:
\[ a(a+b)^* \]

2. Consider the CFG

   \[ S \Rightarrow XaXaX \]
   \[ X \Rightarrow aX|bX|\varepsilon \]

   What is the language this CFG generates?

Solution:
\[ (a+b)^*a(a+b)^*a(a+b)^* \]

3. Find a CFG for each of the languages defined by the following regular expressions:

   a. \[ ab^* \]
   b. \[ a^*b^* \]
   c. \[ (baa + abb)^* \]

Solutions:
   a) \[ S \Rightarrow a | Sb \]
   b) \[ S \Rightarrow XY \]
      \[ X \Rightarrow aX | \varepsilon \]
      \[ Y \Rightarrow bY | \varepsilon \]
   c) \[ S \Rightarrow SS | baa | abb | \varepsilon \]

4. Write a CFG to generate the language MOREA of all strings that have more a’s than b’s.

Solution:
\[ S \Rightarrow SS | EXE \]
\[ X \Rightarrow aX | a \]
\[ E \Rightarrow aB | bA \]
\[ A \Rightarrow a | aS | bAA \]
B -> b | bS | aBB

5. Show that the following CFGs are ambiguous by finding a word with two distinct parse trees.
   i. S -> SaSaS|b
   ii. S -> aSb|Sb|Sa|a.

Solution:
   i) babababab
   ii) aab

6. Build a PDA for the language \{a^mb^na^mb^n | n > 0 and m > 0\}

7. Build a PDA for the language \{a^{2n}b^n | n > 0\}

8. Build a PDA for the language \{a^lb^mc^n | l, m, n > 0 and l + m = n\}