1. All words in which a appears tripled, if at all. This means every clump of a’s contains 3 or 6 or 9 or 12 .... a’s.

\[(aaa+b)^*\]

2. All words that contain exactly two b’s or exactly three b’s, not more.

\[A*ba*ba*(b\varepsilon )a^*\]

3. (i) All strings that end with a double letter.
(ii) All strings that do not end in a double letter.

(i) \[(a+b)^*(aa+bb)\]
(ii) \[(a+b)^*(ab+ba) + a + b + \varepsilon\]

4. All strings that have exactly one double letter in them.

\[(b\varepsilon )(ab)^*aa(ba)^*(b+\varepsilon) + (a+\varepsilon)(ba)^*bb(ab)^*(a+\varepsilon)\]

5. All strings in which the letter b is never tripled. This means that no word contains the substring bbb.

\[(a+ba+bba)^*(bb+b+\varepsilon)\]

6. All words in which a is tripled or b is tripled, but not both. This means each word contains the substring aaa or the substring bbb, but not both.

\[(a+ba+bba)^*(bb+b+\varepsilon)aaa(a+ba+bba)^*(bb+b+\varepsilon) + (b+ab+aab)^*(aa+a+\varepsilon)bbb(b+ab+aab)^*(aa+a+\varepsilon)\]

7. All strings in which the total number of a’s is divisible by 3 no matter how they are distributed, such as aabaabbba.

\[(b*ab*ab*)^*\]

8. Show that the following pairs of regular expressions define the same language over the alphabet \{a b\}

(i) \[(ab)^*a\text{ and } a(ba)^*\]
(ii) \[(a*+b)^*\text{ and } (a+b)^*\]
(iii) \[(a*+b)^*\text{ and } (a+b)^*\]
(i) \((ab)^*a\) consists of all strings \((ab)^n a\) for \(n = 0, 1, \ldots\)
a\((ba)^*\) consists of all strings \(a(ba)^n\) for \(n = 0, 1, \ldots\)

Using induction, we can prove \((ab)^n a = a(ba)^n\) for all \(n\’s\)
1) \((ab)^0 a = a(ab)^0 = a.\)
2) Assume \((ab)^{n-1} a = a(ba)^{n-1}\)
   \((ab)^n a = (ab)^{n-1} aba = ((ab)^{n-1} a)ba = a(ba)^{n-1} ba = a(ba)^n\)

(ii) \((a+b) \subseteq (a^*+b)\), so \((a+b)^* \subseteq (a^*+b)^*\)

Because \((a+b)^*\) contains all strings on \(\{a,b\}\), so \((a^*+b)^* \subseteq (a+b)^*\).

(iii) \((a+b) \subseteq (a^*+b^*)\), so \((a+b)^* \subseteq (a^*+b^*)^*\)

Because \((a+b)^*\) contains all strings on \(\{a,b\}\), so \((a^*+b^*)^* \subseteq (a+b)^*\).

9. Describe in English the languages associated with the following regular expressions.

(i) \((a(a+bb))^*\)

All words that do not begin with \(b\) and in which \(b\’s\) appear in clumps of even lengths.

(ii) \((b(bb)^* (a(aa)^* b(bb)^*)^*\)

all words that start with 0 or more \(b\’s\) followed by odd number \(a\’s\) and \(b\’s\).

(iii) \(((a+b)a)^*\)

all words with even lengths and in which each \(b\) is separated by some \(a\’s\) and a
occupies all even positions.

10. Find a DFA accepting the language \(10+(0+11)0^*1\)