CS5104 Assignment 1
Due: Wednesday, February 11.

1. Consider the language PALINDROME over the alphabet \{a, b\}. Prove that if \(x\) is in PALINDROME, then so is \(x^n\) for any \(n\).

2. Prove that for all sets, \(S\), \((S^+)^+ = S^+\).

3. Give two recursive definitions for the set
   \[ \text{POWER-OF-TWO} = \{1, 2, 4, 8, 16, \ldots\} \]
   Use one of them to prove that the product of two POWER-OF-TWO is also a POWER-OF-TWO.

4. Give recursive definitions for the following languages over the alphabet \{a, b\}:
   a. The language EVENSTRING of all words of even length
   b. The language NOTAA of all words not containing the substring aa.

For problems 5 to 9, assume that the alphabet is \{a, b\}.

5. Build a DFA that accepts only the language of all words with \(b\) as the second letter.

6. Build a DFA that accepts only the words \(baa, ab\), and \(ab\) and no other strings.

7. Build a DFA that accepts only those words that do not end with \(ba\).

8. Build a DFA that accepts only those words that begin or end with a double letter.

9. Build a DFA that accepts only those words that have en even number of substrings \(ab\).

10. Describe in English the languages accepted by the followings DFAs:

    i. 
    
    ![Diagram i]

    ii. 
    
    ![Diagram ii]
11. Construct an NFA that accepts the set of binary strings with at least two occurrences of substring 01 and ends with 11.

12. Convert each of the following NFAs into DFAs.

   i.
   
   ii.